*Astronomy & Astrophysics* manuscript no. hbetarev
April 5, 2006


# Computed $H_\beta$ indices from ATLAS9 model atmospheres

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**ABSTRACT**

*Aims.* Grids of $H_\beta$ indices based on updated (new-ODF) ATLAS9 model atmospheres were computed for solar and scaled solar metallicities [+0.5], [+0.2], [−0.5], [0.0], [−1.0], [−1.5], [−2.0], [−2.5] and for $\alpha$ enhanced compositions [+0.5a], [0.0a], [−0.5a], [−1.0a], [−1.5a], [−2.0a], [−2.5a], and [−4.0a].
*Methods.* Indices for $T_{\rm eff}$>5000 K were computed with the same methods as described by Lester, Gray & Kurucz (1986)(LGK86) except for a different normalization of the computed natural system to the standard system. LGK86 used special ODFs to compute the fluxes. For $T_{\rm eff}$≤5000 K we computed the fluxes using the synthetic spectrum method. In order to assess the accuracy of the computed indices comparisons were made with the indices computed by Smalley & Dworetsky (1995)(MD95) and with the empirical relations $T_{\rm eff}$-$H_\beta$ given by Alonso et al. (1996) for several metallicities. Furthermore, for cool stars, temperatures inferred from the computed indices were compared with those of the fundamental stars listed by MD95. The same kind of comparison was made between gravities for B-type stars.
*Results.* The temperatures from the computed indices are in good agreement, within the error limits, with the literature values for 4750 K≤$T_{\rm eff}$≤8000 K, while the gravities agree for $T_{\rm eff}$>9000 K. The computed $H_\beta$ indices for the Sun and for Procyon are very close to the observed values. The comparison between the observed and computed $H_\beta$ indices as function of the observed $H_\beta$ has shown a very small trend which almost completely disappears when only stars hotter than 10000 K are considered. The trend due to the cool stars is probably related with the low accuracy of the fundamental $T_{\rm eff}$ which are affected by large errors for most of the stars.

**Key words.** Stars: atmospheres – Stars: fundamental parameters–Astronomical techniques: photometric

## 1. Introduction

$H_\beta$ indices are an useful tool to derive stellar parameters, in particular the temperature for A4-G8 stars ($T_{\rm eff}$∼8000-5500 K), but especially the gravity for B-type stars ($T_{\rm eff}$∼30000-10500 K).

Previous last works on synthetic $H_\beta$ indices from ATLAS9 models are those from Smalley & Dworetsky (1995) (hereafter SD95) and Lester, Gray & Kurucz (1986) (hereafter LGK86). SD95 computed the indices only for the solar metallicity, while LGK provided them for numerous different metallicities.

Both LGK86 and SD95 used the same ATLAS9 models from Kurucz (1979), but they adopted different filter curves and different methods to normalize the computed indices to the observed indices. LGK86 adopted as transmission curves the filter set (212,214) from Crawford & Mander (1966) and selected five stars ($\gamma$ Gem (HD 47105), $\alpha$ CMi (HD 61421), $\beta$ Leo (HD 102647), $\eta$ UMa (HD 120315), and $\alpha$ Lyr (HD 172167)) to fix the transformation from the natural system of the computed indices to the standard system. SD95 used two Gaussian response functions with widths of 30 Å and 150 Å, respectively. They derived spectroscopic $H_\beta$ indices from spectra of more than 50 stars observed at medium resolution. The normalization of the spectroscopic indices to the standard system was performed by fitting the spectroscopic indices to the photometric indices. The same procedure was used to obtain synthetic $H_\beta$ indices that they derived by applying the filters to synthetic spectra centered on the H$\beta$ profile.

Castelli (1991) and Napiwotzki et al. (1993) found that the LGK86 indices were not accurate enough for stars hotter than about 15000 K and indicated the cause of this shortcoming in the values adopted by LGK86 for the model parameters of $\eta$ UMa. Vice-versa, SD95 showed that their synthetic indices closely agree with the photometric indices for stars covering a large range of temperatures and gravities. However, their computed indices are limited only to the solar metallicity.

The $H_\beta$ indices presented in this paper are based on updated ATLAS9 models atmospheres (new-ODF models, Castelli & Kurucz 2003). The indices were computed both for scaled solar metallicities and scaled solar metallicities with an enhancement of +0.4 dex for the $\alpha$ element abundances. The metallic-

*Send offprint requests to*: F. Castelli ity ranges from [-2.5] to [+0.5], the temperature from 3500 K to 50000 K and the gravity from log $g$=0.0 to log $g$=5.0.

As described in Sect. 2, we used the same response functions as LGK86, but we adopted a different method to scale the computed indices to the observed indices.

To investigate the accuracy of the synthetic $H_\beta$ indices we compared the $T_{\rm eff}$-$H_\beta$ relations from our indices with those from MD95 amd LGK86, and, for cool stars, with the empirical relations from Alonso et al. (1996). Also the difference of observed and computed $H_\beta$ indices was plotted as function of the observed $H_\beta$. Finally, we compared stellar parameters $T_{\rm eff}$ and log $g$ inferred from $H_\beta$ with the parameters of fundamental stars as they were determined by SD95 and successive updated papers.

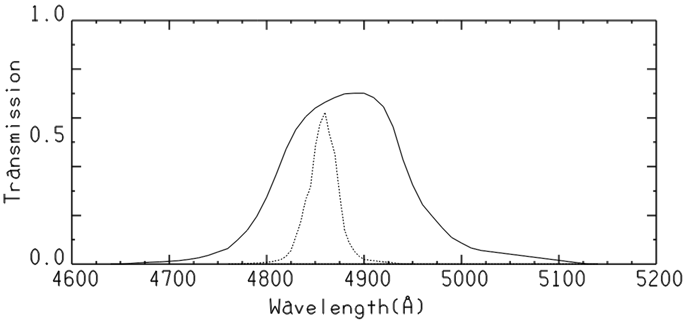


**Fig. 1.** Transmission curves for the filter set (212,214) from Crawford & Mander (1966).The dotted line is the narrow filter 212 and the full line is the wide filter 214.

## 2. Synthetic $H_\beta$ indices

The $H_\beta$ index is defined as the magnitude difference corresponding to the ratio of the intensities measured through two interference filters centered on $H_\beta$ with half-widths of about 30 Å and about 150 Å, respectively. By means of the HBETA code from Kurucz[1] we computed the quantity:

$$H'_\beta = -2.5[\log \frac{\int_{\alpha_N}^{\beta_N} F_\lambda S_N(\lambda) d\lambda}{\int_{\alpha_N}^{\beta_N} S_N(\lambda) d\lambda} - \log \frac{\int_{\alpha_W}^{\beta_W} F_\lambda S_W(\lambda) d\lambda}{\int_{\alpha_W}^{\beta_W} S_W(\lambda) d\lambda}]$$

which can be considered as the index in the natural system. $S_N(\lambda)$ and $S_W(\lambda)$ are the response functions of the narrow and wide filters respectively, and $F_\lambda$ is the computed stellar flux at the star surface.

### 2.1. The response functions $S_N(\lambda)$ and $S_W(\lambda)$

The adopted transmission curves are the Kitt Peak filter set (212,214) from Crawford & Mander (1966) (Fig. 1). If both filters were symmetric on exactly the same central wavelength, $H_\beta$ would be free from interstellar reddening and also from atmospheric extinction, provided that simultaneous measurements through the two filters are performed. But the two filters do not have the same central wavelength, which is 4859 Å for the narrow filter and 4890 Å for the wide filter. Therefore, they were corrected in the computations for the atmospheric extinction, for the response function of a standard 1 P21 photomultiplier, and for the reflectivity of aluminium (Kurucz 1979). Because all these data are either scattered in the literature or are available only inside Kurucz's programs we have collected them in Appendix A, Table A.1.

### 2.2. The computed fluxes $F_\lambda$

Fluxes $F_\lambda$ in the $H_\beta$ region were computed for $T_{\rm eff}\geq$5250 K by means of the BETA9 code of Kurucz which adds the contribution of an accurately computed H$\beta$ profile (Peterson's subroutine HPROF4 of the SYNTHE[2] code) to the line blanketing yielded by special ODFs. Wavelengths for H$\beta$ were chosen spreading away from line center at 4861.32 Å to give good resolution both to the line and to the two filters. The special ODF's were computed by Kurucz who used all the atomic lines, except H$\beta$, lying in the region 4700−5200 Å. The line blanketing was tabulated at 10 Å intervals for 56 values of temperature T and 21 values of gas pressure $P_{gas}$. The special ODFs are available for several metallicities at the Kurucz website[3]. They are VMS binary files which have names BDFxxxBETA.DAT, where xxx indicates the metallicity. Each file contains 5 ODFs computed for microturbulent velocities $\xi$=0.0, 1.0, 2.0, 4.0 and 8.0 km s$^{-1}$. We have converted the binary ODFs in ASCII format in order to use them with whichever available operating system. In particular, we computed $H_\beta$ indices under GNU-Linux.

Because no molecular lines were included in the special ODFs computed by Kurucz, fluxes from ODFs were replaced by fluxes from synthetic spectra for $T_{\rm eff}\leq$5000 K. The fluxes were computed with the SYNTHE code (footnote 2) at 500000 resolution using both atomic[4] and molecular[5] Kurucz line lists. No atomic lines from predicted levels were considered.

### 2.3. Normalization of the $H_\beta$ index

For the Kitt Peak filters (212,214) the relation between the natural system and the standard system is not unique for all the stars, but there are two different relations for B and for A, F spectral types (Crawford & Mander,1966). Therefore, we transformed the $H'_\beta$ indices to the standard system by means of the relations given by Crawford & Mander (1966):

$H_\beta = 0.374 + 1.305 H'_\beta$ for B type stars
$H_\beta = 0.248 + 1.368 H'_\beta$ for A, F type stars

As further step, the two $H_\beta$ indices computed for Vega by using the two above relations were forced to match the observed index $H_\beta$=2.904 (Crawford et al. 1972). Namely, the constants 2.258 and 2.151 were added to $H_\beta$(Vega)=0.753 obtained from the first relation and to $H_\beta$(Vega)=0.646 obtained from the second relation. Because we assumed $T_{eff}$=10500 K as limit between B and A, F spectral types we added 2.258

[1] program beta.forcd in http://kurucz.harvard.edu/programs/COLORS
[2] http://kurucz.harvard.edu/programs/SYNTHECD/synthe.for
[3] http://kurucz.harvard.edu/OPACITIES/DFBETA
[4] http://kurucz.harvard.edu/LINELISTS/GF100
[5] http://kurucz.harvard.edu/molecules.html

to all the indices computed for $T_{\rm eff}$≤10500 K, and 2.151 to all those computed for $T_{\rm eff}$>10500 K. We experimented to determine which are the computed indices for Vega if the integrals over the filters (i.e. the denominators in the formula for $H'_\beta$) are dropped. We found that they are the same. They become $H_\beta$=3.000 and $H_\beta$=2.999 for B and for A, F stars, respectively, so that the constants −0.096 and −0.095 have to be added to all the computed $H_\beta$ indices. However, the final result is the same does not matter whether the integrals over the filters at the denominator are considered or not in the $H'_\beta$ computations.

The Vega model used to compute $H_\beta$ is a new-ODF ATLAS9 model (Castelli & Kurucz 2003) with parameters $T_{eff}$=9550 K, logg=3.95, $\xi$=2 km s$^{-1}$, [M/H]=−0.5 (Castelli & Kurucz 1994).

The resulting index for the Sun is $H_\beta$=2.590, in good agreement with $H_\beta$=2.5955±0.0005 observed by Olsen (1976) and 2.591±0.005 determined by Saxner & Hammarback (1985). The model adopted for the Sun is a new-ODF ATLAS9 model (Castelli & Kurucz 2003) with parameters $T_{eff}$=5777 K, log g=4.44377, $\xi$=1.0 km s$^{-1}$, and [M/H]=0.0. The SD95 grid gives $H_\beta$=2.581 for the Sun.

For $\eta$ UMa, Napiwotzki et al. (1993) found that the log $g$ which best fits both $H_\beta$ and $H_\gamma$ profiles for $T_{\rm eff}$=17000 K is log $g$=4.24. Interpolating for the same temperature and gravity in our $H_\beta$ grid computed for [M/H]=0.0 and $\xi$=2 km s$^{-1}$ we obtain $H_\beta$= 2.689 which differs by −0.05 dex from the observed value 2.694±0.001 (Hauck & Mermilliod 1998)[6]. SD95 obtained $H_\beta$=2.698, which differs by +0.04 dex from the observed value. Vice-versa, for $T_{\rm eff}$=17000 K we derived log $g$=4.31 from the observed $H_\beta$ index, while SD95 yield log $g$=4.18.

## 3. The computed grids of $H_\beta$ index

$H_\beta$ indices were computed as a function of $T_{\rm eff}$ and log $g$ for the microturbulent velocity $\xi$=2.0 km s$^{-1}$ for the following metallicities: [+0.5], [+0.5a], [+0.2], [0.0], [0.0a], [−0.5], [−0.5a], [−1.0], [−1.0a], [−1.5], [−1.5a], [−2.0], [−2.0a], [−2.5], [−2.5a], [−4.0a]. A further table of $H_\beta$ indices was computed for [M/H]=0.0 and $\xi$=0.0 km sec$^{-1}$. The suffix "a"means that the abundances of the $\alpha$ elements, O, Ne, Mg, Si, S, Ar, Ca, and Ti are enhanced by +0.4 dex over the solar or scaled-solar abundances. The new-ODFs grids of model atmospheres (Castelli & Kurucz, 2003) were used as input models for the computations. There are 476 models for each metallicity. The model parameters of the grids are given in Table 1 of Castelli & Kurucz (2003). These models differ from the Kurucz (1979) model atmospheres for the atomic line blanketing which was computed with a much larger number of lines in the new models and with the addition of the molecular line opacity which was completely left out in the 1979 models. Furthermore, updated solar abundances (Grevesse & Sauval 1998), updated continous opacities, and 72 atmospheric layers instead of 64, were adopted.

6 http://obswww.unige.ch/gcpd/gcpd.html

**Table 1.** Example of table with computed $H_\beta$ indices

| Teff | log g | [M] | Vturb | l/H | small | big | beta |
|---|---|---|---|---|---|---|---|
| Teff | log g | [M] | Vturb | l/H | small | big | beta |
| 3500 | 0.00 | 0.00 | 2.00 | 1.25 | -12.354 | -12.505 | 2.713 |
| 3500 | 0.50 | 0.00 | 2.00 | 1.25 | -12.321 | -12.493 | 2.742 |
| 3500 | 1.00 | 0.00 | 2.00 | 1.25 | -12.286 | -12.477 | 2.768 |
| 3500 | 1.50 | 0.00 | 2.00 | 1.25 | -12.269 | -12.476 | 2.789 |
| 3500 | 2.00 | 0.00 | 2.00 | 1.25 | -12.281 | -12.498 | 2.803 |
| 3500 | 2.50 | 0.00 | 2.00 | 1.25 | -12.327 | -12.549 | 2.810 |
| 3500 | 3.00 | 0.00 | 2.00 | 1.25 | -12.414 | -12.634 | 2.807 |
| 3500 | 3.50 | 0.00 | 2.00 | 1.25 | -12.551 | -12.757 | 2.788 |
| 3500 | 4.00 | 0.00 | 2.00 | 1.25 | -12.744 | -12.919 | 2.746 |
| 3500 | 4.50 | 0.00 | 2.00 | 1.25 | -12.932 | -13.069 | 2.693 |
| 3500 | 5.00 | 0.00 | 2.00 | 1.25 | -13.074 | -13.175 | 2.644 |
| 3750 | 0.00 | 0.00 | 2.00 | 1.25 | -13.602 | -13.660 | 2.586 |
| 3750 | 0.50 | 0.00 | 2.00 | 1.25 | -13.633 | -13.689 | 2.583 |
| 3750 | 1.00 | 0.00 | 2.00 | 1.25 | -13.637 | -13.693 | 2.583 |
| 3750 | 1.50 | 0.00 | 2.00 | 1.25 | -13.623 | -13.683 | 2.588 |
| 3750 | 2.00 | 0.00 | 2.00 | 1.25 | -13.600 | -13.666 | 2.596 |
| 3750 | 2.50 | 0.00 | 2.00 | 1.25 | -13.577 | -13.649 | 2.606 |
| 3750 | 3.00 | 0.00 | 2.00 | 1.25 | -13.566 | -13.645 | 2.614 |
| 3750 | 3.50 | 0.00 | 2.00 | 1.25 | -13.577 | -13.659 | 2.618 |
| 3750 | 4.00 | 0.00 | 2.00 | 1.25 | -13.625 | -13.700 | 2.609 |
| 3750 | 4.50 | 0.00 | 2.00 | 1.25 | -13.706 | -13.766 | 2.588 |
| 3750 | 5.00 | 0.00 | 2.00 | 1.25 | -13.794 | -13.837 | 2.564 |
| 4000 | 0.00 | 0.00 | 2.00 | 1.25 | -14.350 | -14.405 | 2.581 |

Tables of $H_\beta$ indices for all the above quoted metallicities are availabe on our website[7]. They have the form shown in Table 1, where the columns 6 and 7 give the magnitude through the small and big filter, respectively, at the star surface. The last column lists the final $H_\beta$ indices after normalization to Vega. The full table available on the website extends from $T_{\rm eff}$=3500 K to $T_{\rm eff}$=50000 K.

## 4. Dependence of $H_\beta$ on $T_{\rm eff}$, log $g$, and metallicity

The dependence of $H_\beta$ on $T_{\rm eff}$ for several different gravities is shown in Fig.2 for [M/H]=0.0 and $\xi$=2 km s$^{-1}$. $H_\beta$ is a good gravity index for both giants and dwarfs for $T_{\rm eff}$≥9250 K provided that $T_{\rm eff}$ is known rather well in advance. The lower limit of 9250 K can be progressively shifted toward lower temperatures as the gravity lowers.

The two different normalizations of the indices for $T_{\rm eff}$≤10500 K and $T_{\rm eff}$>10500 K give rise to a small discontinuity of the index at 10500 K for gravities lower than 3.5 dex. The discontinuity was reduced by smoothing $H_\beta$ over $T_{\rm eff}$ from 10250 K to 11000 K. For [M/H]=0.0 and log $g$=2.0 the indices at 10500 K and 10750 K are respectively 2.611 and 2.618 without any smoothing and 2.618 and 2.615 after smoothing. At $T_{\rm eff}$=10500 K the difference between the indices obtained with and without smoothing decreases from 0.007 dex for log $g$=2.0 to 0.001 dex for log $g$=5.0. We note that the only effect of a different choice for the temperature limit between the two different calibrations is a shift of the discontinuity to the position of the new $T_{\rm eff}$. This problem, affecting the indices computed

7 http://wwwuser.oat.ts.astro.it/castelli/colors/hbeta.html

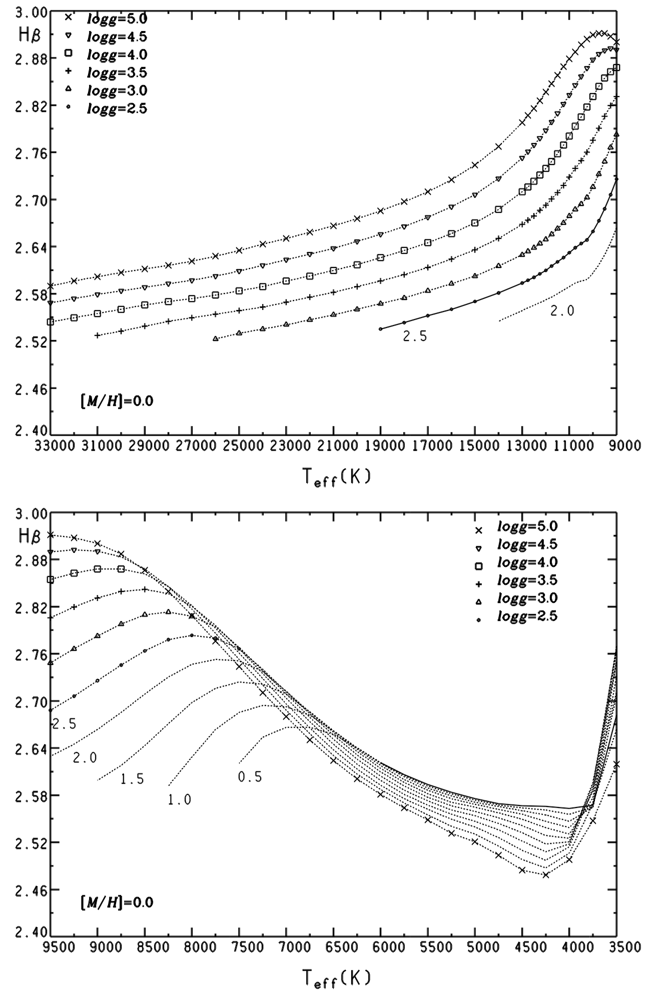


**Fig. 2.** The dependence of the $H_\beta$ index on $T_{\rm eff}$ for different gravities, [M/H]=0.0 and $\xi$=2 km s$^{-1}$

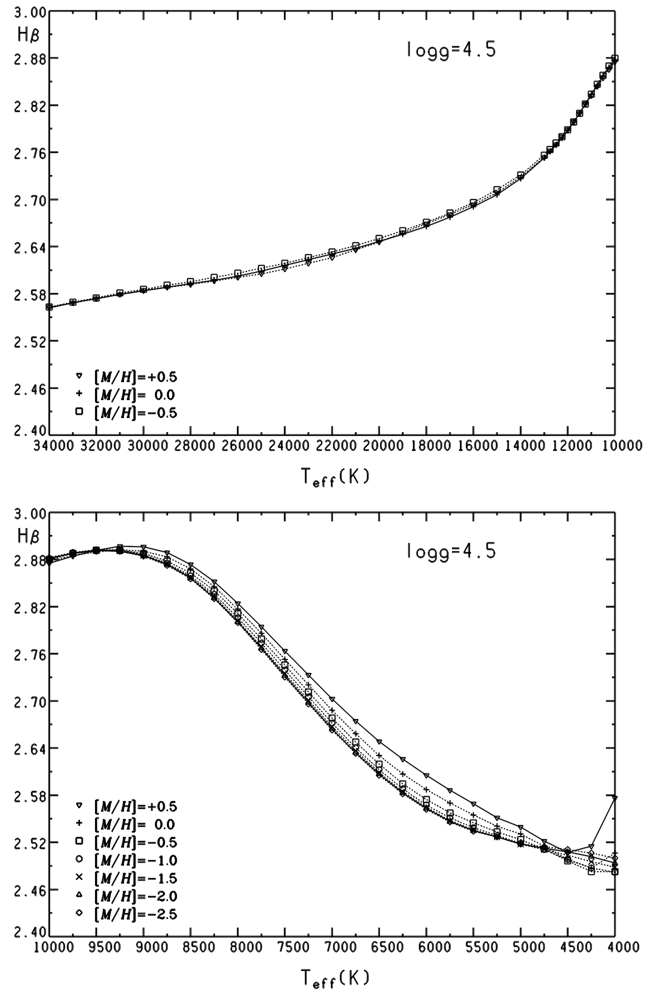


**Fig. 3.** The dependence of the $H_\beta$ index on $T_{\rm eff}$ for different metallicities for log $g$=4.5

for low gravities, is possibly related with the very small number of giants that were used by Crawford & Mander (1966) to fix the two transformation equations from the natural system to the standar system for B and A,F stars.

$H_\beta$ can be used as a temperature index for dwarfs (log $g \geq 3.0$) having $T_{\rm eff}$ between 8000 K and 4500 K and for giants (log $g$<3.0) with $T_{\rm eff} \geq 4750$ K, but with a decreasing of the upper limit from 7500 K for log $g$=2.5 to 6500 K for log $g$=0.5. As noted by Alonso et al. (1996), $H_\beta$ is a poorer temperature indicator than most of the other photometric colours, but because it is almost independent from interstellar extinction it may be useful to derive temperatures for reddened stars, although below about $T_{\rm eff}$=4500 K it is meaningless because rather than indicate the H$\beta$ strength it measures the line blanketing due the numerous lines lying in the H$\beta$ region, which may be also stronger than the H$\beta$ line itself.

Figure 3 shows the dependence of $H_\beta$ on metallicity for log $g$=4.5. The effect of the metallicity is fully negligible for $T_{\rm eff} \geq 9250$ K, while there is some spread for cooler temperatures; but the spread decreases with decreasing metallicity and disappears when [M/H] $\leq -1.0$. We checked that this behaviour is similar also for other gravities.

The comparison between $H_\beta$ indices computed for [M/H]=0.0 and different microturbulent velocities $\xi$=2 km s$^{-1}$ and $\xi$=0.0 km s$^{-1}$ has shown fully negligible differences.

## 5. Comparison with other $H_\beta$ determinations

Alonso et al. (1996) calibrated $T_{\rm eff}$ versus the $H_\beta$ index for main sequence stars with temperatures between 4000 K and 7000 K and for metallicities from [+0.5] to [−3.5]. Figure 4 compares the empirical $T_{\rm eff}$-$H_\beta$ relations taken from Table 3 of Alonso et al. (1996) for [M/H]=0.0, −1.0, −2.0, and −2.5 with the corresponding computed relations for log $g$=4.5. Both the empirical relations for [M/H]=−2.0 and [M/H]=−2.5 and the two corresponding computed relations are undistinguishable for $T_{\rm eff}$>5000 K. Figure 4 shows agreement between computed and empirical relations for $T_{\rm eff} \leq 4250$ K for [M/H]=0.0, while the computed indices are larger by several magnitudes than the observed indices for the lower metallicities when $T_{\rm eff}$< 5000 K.

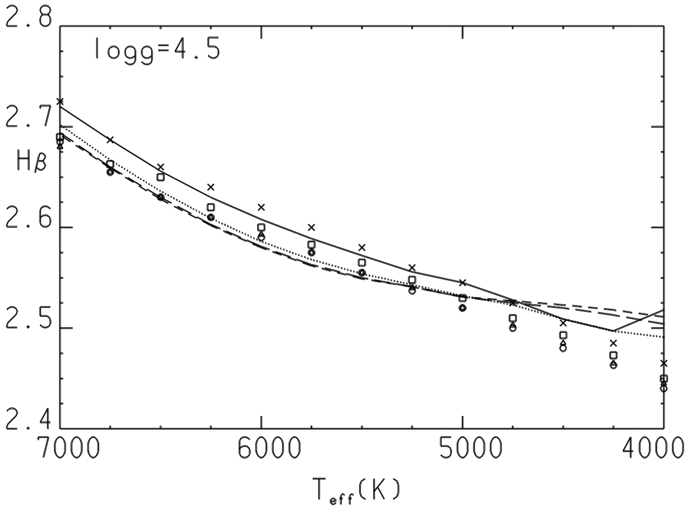


**Fig. 4.** Empirical relations $T_{\rm eff}$-$H_\beta$ from Alonso et al. (1996) for F0V-K5V stars for [M/H]=0.0 (crosses), −1.0 (squares), −2.0 (triangles), and −2.5 (circles) ) are compared with the computed relations for log $g$=4.5 and [M/H]=0.0 (full line), −1.0 (dotted line), −2.0 (long-dashed line), −2.5 (dashed line).

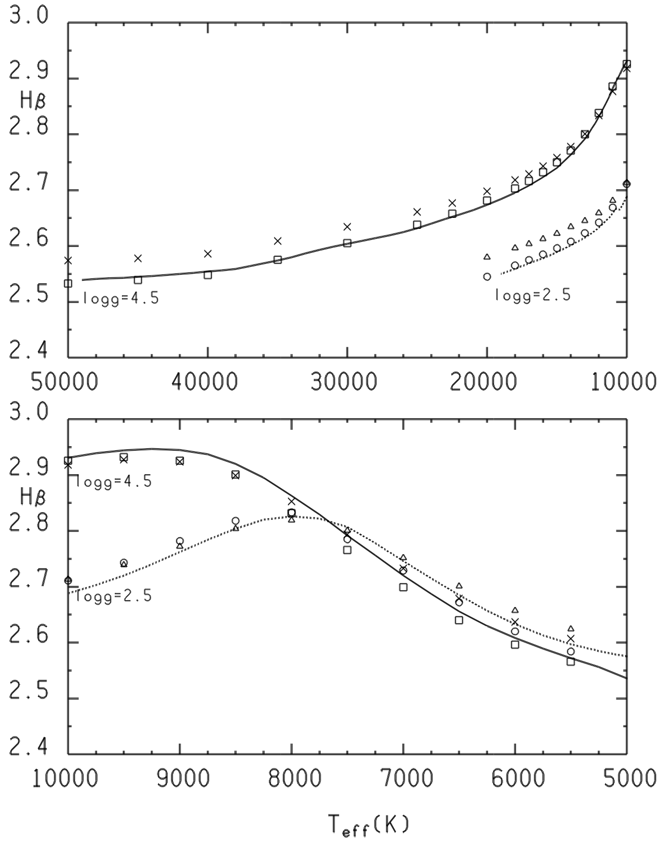


**Fig. 5.** Comparison of $H_\beta$ indices from this paper (full line for log $g$=4.5, dotted line for log $g$=2.5) with $H_\beta$ indices from LGK86 (crosses for log $g$=4.5, triangles for log $g$=2.5) and SD95 (squares for log $g$=4.5, circles for log $g$=2.5).

We note that both observed and computed indices have to be taken with caution at these temperatures.

For $T_{\rm eff}$≥4750 K, the largest discrepancy between the computed and empirical relations occurs at 5750 K, where for [M/H]=0.0, the difference $T_{\rm eff}$(empirical)-$T_{\rm eff}$(this paper) is −145 K. This behaviour is similar for the other metallicities. This maximus temperature difference of −145 K is very close to the uncertainty ΔT=141 K assigned by Alonso et al. (1996) to the $T_{\rm eff}$-$H_\beta$ empirical relation.

Figure 5 compares the computed $T_{\rm eff}$-$H_\beta$ relations from this paper for [M/H]=0.0, $\xi$=2 km s$^{-1}$ and both log $g$=4.5 and log $g$=2.5 with those inferred from the SD95 and the LGK86 computed indices. There is a close agreement with the SD95 relation, especially for hot stars. For cool stars, for a given $H_\beta$, the SD95 temperatures are a little bit higher than those from this paper. The difference $T_{\rm eff}$(SD95)−$T_{\rm eff}$(this paper) increases from 94 K at 5500 K to 228 K at 8000 K. As a consequence, the discrepancy between $T_{\rm eff}$-$H_\beta$ relation from SD95 and the empirical relation from Alonso et al. (1996) is still larger than that we obtained.

For hot stars the difference in $H_\beta$ for a given temperature corresponds to a difference in log $g$ not larger than 0.12 dex which occurs at 15000-16000 K for log $g$=4.5. Our indices predict higher gravities than SD95.

For a given $H_\beta$ the LGK indices give lower temperatures for cool stars than those from both our indices and the SD95 indices and give higher gravities for hot stars. This discrepancy increases with increasing temperature, so confirming the trend shown by Castelli (1991).

## 6. Stellar parameters from the computed $H_\beta$ indices

We compared $T_{\rm eff}$ and log $g$ that we derived from the computed $H_\beta$ index with the parameters obtained by SD95 (and further improvements) for a set of stars called fundamental stars according to Hayes (1978), in that their model parameters were fixed by means of model-independent methods (fundamental methods) (see also Smalley 2005).

Among the stars studied by SD95 there are only four stars (HD 16739, HD 40183, HD 110379, and HD 202275) with both $T_{\rm eff}$ and log $g$ given as fundamental parameters, while for all the other stars only either $T_{\rm eff}$ or log $g$ is a fundamental value. For these stars the other non-fundamental parameter was derived spectroscopically by SD95 from H$\beta$ profiles and/or from the spectrophotometry. Stars with fundamental $T_{\rm eff}$ are the Code et al. (1976) stars reanalyzed by SD95. Stars with fundamental log $g$ are binary stars. Successively, for a few stars the parameters were improved by Gardiner et al. (1999) and especially by Smalley et al. (2002), who derived fundamental values of $T_{\rm eff}$ for several binary stars of the SD95 sample. We adopted for our comparisons the last values for the parameters that we found in the above quoted series of papers. The sources

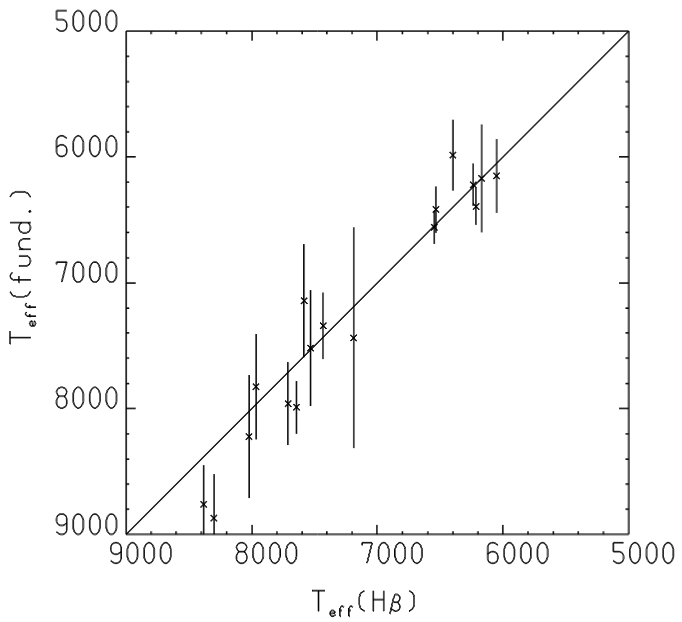


**Fig. 6.** $T_{\rm eff}$ computed from the $H_\beta$ index is compared with $T_{\rm eff}$ taken from the SD95 sample (see Table2) of stars having one of both parameters given as fundamental. The error bars are the errors in $T_{\rm eff}$ from the as taken from the literature sources listed in Table 2, col. 9.

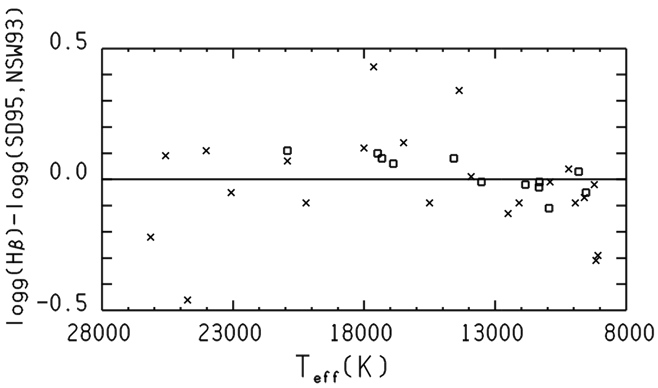


**Fig. 7.** $\Delta$log $g$ differences between the gravities derived from the $H_\beta$ index and those taken from the literature (see Table 3) are plotted as function of $T_{\rm eff}$. Squares indicate log $g(H_\beta)-$log $g$(NSW93) from Table 3. Crosses indicate all the others $\Delta$log $g$ from Table 3, col.8.

are listed in col. 9 of Table 2 and Table 3. The fundamental parameter is indicated in the last column of the tables.

Table 2 compares $T_{\rm eff}$ that we derived from the $H_\beta$ index with $T_{\rm eff}$ taken from the above quoted literature for stars cooler than 9000 K. Figure 6 shows that for most of the stars with $T_{\rm eff}\leq$8000 K the temperatures inferred from $H_\beta$ agree with the literature values, within the error limits (actually, very large for some stars). Exceptions are HD 90242 and HD 187642 which lie outside the error limits. We can note the excellent agreement for Procyon (HD 61421) which has the most tightly constrained value of $T_{\rm eff}$ due to the accuracy of the direct measurement of angular diameter. The poor agreement for the two hottest stars with $T_{\rm eff}>$8000 K, HD 102647 ($T_{\rm eff}$(fundamental)=8870 K) and HD 216956 ($T_{\rm eff}$(fundamental)=8760 K) can be explained with the overcoming of the validity limits for $H_\beta$ as temperature index (fig. 2).

Table 3 compares log $g$ that we derived from the $H_\beta$ index with log $g$ of the MD95 sample for stars hotter than 9000 K. There is only a star with an exceptionally large disagreement. It is HD 169022 for which $\Delta$log $g$ is $-$1.61. Other stars with large $\Delta$log $g$ have the gravity derived by MD95 from the spectrophotometry, a method giving values affected by errors of the order of 0.5-0.3 dex. We added in Table 3 the comparison of log $g$ from $H_\beta$ with log $g$ determined from Balmer profiles by Napiwotzki et al. (1992) (hereafter NSW93) for 12 B-type stars. These are not fundamental stars having $T_{\rm eff}$ derived from observed Strömgren indices and the TEFFLOGG code of Moon (1985). We adopted the temperatures from NSW93. Note that Napiwotzki et al. (1993) adopted for $\eta$ UMa both $T_{\rm eff}$=17000 K and $T_{\rm eff}$=17320 K to derive log $g$ from the Balmer profiles (their Fig.18 and Table 4). For comparison purposes we used the two different $T_{\rm eff}$ values here and in Sect. 2.3. The agreement is within 0.11 dex for all the stars. Excluding HD 169022, Fig. 7 shows that the gravity derived from the $H_\beta$ index agrees, on average, with the literature gravity obtained from other methods.

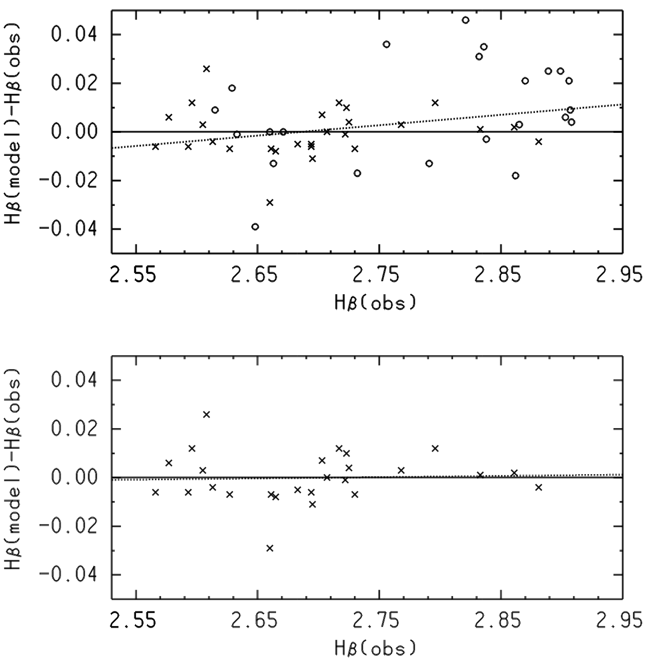


**Fig. 8.** Comparison between the observed $H_\beta$ indices and those predicted for the stars listed in Table 2 and Table 3 as function of the observed $H_\beta$. In the upper plot all the stars (except $\epsilon$ Sgr) are plotted. Stars hotter than 10000 K are shown as crosses, the cooler ones as open circles. The dashed line is the least squares fit to all the points. In the lower plot only stars with $T_{\rm eff}>$10000 K are plotted. The trend disappears almost completely.

**Table 2.** $T_{\rm eff}$ (col. 7) from the observed $H_\beta$ index (col. 6) is compared with $T_{\rm eff}$ (col. 4) of stars having either $T_{\rm eff}$ (col. 4) or log $g$ (col. 5) or both determined as fundamental values and taken from the literature (sources in col. 9). Last column indicates which are the fundamental parameters.

| HR | star | HD | $T_{\rm eff}$ | log $g$[1] | $H^2\beta$(obs) | $T_{\rm eff}(H_\beta)$ | $\Delta T_{\rm eff}$ | source[3] | fundamental |
|---|---|---|---|---|---|---|---|---|---|
| 788 | 12 Per | 16739 | 6220±168 | 4.20±0.03 | 2.633 | 6237 | 17 | 1 | $T_{\rm eff}$,log $g$ |
| | UX Men | 37513 | 6151±293 | 4.29±0.01 | 2.615 | 6050 | −101 | 1 | $T_{\rm eff}$,log $g$ |
| 2326 | $\alpha$ Car | 45348 | 7520±460 | 1.0 (<1.5ph) | 2.732 | 7533 | 13 | 2 | $T_{\rm eff}$ |
| 2373 | WW Aur | 46052 | 7827±419 | 4.17±0.02 | 2.862 | 7967 | 140 | 1 | $T_{\rm eff}$,log $g$ |
| 2693 | $\delta$ CMa | 54605 | 6170±430 | 1.0 (<1.00ph) | 2.660±0.001 | 6170 | 0 | 2 | $T_{\rm eff}$ |
| 2943 | $\alpha$ CMi | 61421 | 6560±130 | 4.06±0.06 | 2.671±0.000 | 6564 | 4 | 3 | $T_{\rm eff}$,log $g$ |
| 3524 | RS Cha | 75747 | 7342±266 | 4.00±0.21 | 2.791±0.002 | 7431 | 89 | 1 | $T_{\rm eff}$,log $g$ |
| | HS Hya | 90242 | 5985±282 | 4.34±0.006 | 2.648±0.003 | 6400 | 415 | 1 | $T_{\rm eff}$,log $g$ |
| 4534 | $\beta$ Leo | 102647 | 8870±350 | 4.1 (4.0±0.25b,4.1±0.3ph) | 2.899±0.002 | 8302 | −568 | 2 | $T_{\rm eff}$ |
| 4825 | $\gamma$ Vir | 110379 | 7143±450 | 4.21±0.017 | 2.809 | 7582 | 439 | 1 | $T_{\rm eff}$,log $g$ |
| 6556 | $\alpha$ Oph | 159561 | 7960±330 | 3.80±0.3ph | 2.832 | 7709 | −251 | 3 | $T_{\rm eff}$ |
| 6611 | V624 Her | 161321 | 8222±489 | 3.93±0.014 | 2.870±0.003 | 8021 | −201 | 1 | $T_{\rm eff}$,log $g$ |
| 7484 | V1143 Cyg | 185912 | 6418±184 | 4.32±0.02 | 2.663 | 6533 | 115 | 1 | $T_{\rm eff}$,log $g$ |
| 7557 | $\alpha$ Aql | 187642 | 7990±210 | 4.00±0.3ph | 2.821±0.011 | 7643 | −347 | 4 | $T_{\rm eff}$ |
| | MY Cyg | 193637 | 7437±877 | 4.01±0.02 | 2.756±0.002 | 7189 | −248 | 1 | $T_{\rm eff}$,log $g$ |
| 8123 | $\delta$ Equ | 202275 | 6393±147 | 4.34±0.02 | 2.629±0.006 | 6214 | −179 | 1 | $T_{\rm eff}$,log $g$ |
| 8728 | $\alpha$ PsA | 216956 | 8760±310 | 4.20±0.3ph | 2.906±0.004 | 8382 | −378 | 3 | $T_{\rm eff}$ |

[1] "ph" indicates gravities derived from the spectrophotometry; "b" indicates gravities derived from Balmer profiles.
[2] Hauck& Mermilliod (1998) (see also footnote 6 in the text).
[3] Sources for $T_{\rm eff}$ and log $g$ in cols. 4 and 5:
(1) Smalley et al. (2002) (Setal02); (2) Smalley & Dworetsky (1995) (SD95); (3) Gardiner et al. (1999) (GKS99); (4) Moon & Dworetsky (1985) (MD85).

Figure 8 shows a comparison between the observed $H_\beta$ indices given Table 2 and Table 3 (Hauck & Mermilliod (1998) and footnote 6) and those interpolated in the grid computed for [M/H]=0.0, $\xi$=2.0 Km s$^{-1}$, by assuming for each star $T_{\rm eff}$ and log $g$ given in cols. 4 and 5 of Table 2 and cols. 4 and 6 of Table 3. On average, the differences are larger for the stars cooler than $T_{\rm eff}$=10000 K than for the hotter stars. This is due to the large uncertainties in $T_{\rm eff}$ that affect the fundamental determination of this parameter. There is a small trend mostly due to the cooler stars. In fact, the trend disappears almost completely if only stars hotter than 10000 K are considered. In this case the difference between computed and observed indices averages to zero.

## 7. Conclusions

We have presented synthetic $H_\beta$ indices based on new-ODF ATLAS9 model atmospheres (Castelli & Kurucz, 2003). The comparison with other determinations and with the observed indices of fundamental stars has shown that they are comparable or even superior to those computed by Smalley & Dworetsky (1995), which were the best ones from ATLAS9 available up to now.

For cool main sequence stars, the computed $T_{\rm eff}$-$H_\beta$ relation agrees better with the empirical relation from Alonso et al. (1996) than the SD95 indices do. In fact, for a given $H_\beta$ and $T_{\rm eff}\geq$5000 K, our indices are the same or a little bit bluer (with a maximus difference of 146 K at 6000 K) than those from Alonso et al., but the SD95 indices are still more bluer. We note that our indices (except at 6000 K) are within the uncertainty of 141 K of the $T_{\rm eff}$-$H_\beta$ empirical relation. For $T_{\rm eff}$<5000 K, the agreement between the computed and the empirical relations extends up to $T_{\rm eff}$=4250 K for solar metallicity, but it is remarkably reduced for lower metallicities. Because $T_{\rm eff}$-$H_\beta$ empirical relations are only estimates for sub solar metallicities when $T_{\rm eff}\leq$ 4500 K and because the utility of the $H_\beta$ index is very scarce in this region of the H-R diagram we did not worry to further investigate about these discrepancies.

We showed in Sect. 2.3 that for the Sun the observed $H_\beta$ is better reproduced by our computations than by those of SD95. Also the computed index for Procyon ($\alpha$ CMi) for $T_{\rm eff}$=6560 K, log $g$=4.06 is the same as the observed one, $H_\beta$=2.671±0.000, while SD95 derived $H_\beta$=2.654. The large differences in $T_{\rm eff}$ for some fundamental cool stars obtained by us from the comparison of the computed and observed indices are very probably related with the low accuracy of the fundamental $T_{\rm eff}$ which is affected by large errors (Table 2).

For hot stars ($T_{\rm eff}$>9000 K), the trend of the differences $\Delta\beta$ between the computed and observed indices as function of $H_\beta$(obs) is fully negligible. The comparison of our indices with the SD95 ones has shown a close agreement, although the latter ones would generally give a little bit lower gravity. An example is $\eta$ UMa that we discussed in Sect. 2.3.

In this paper we have performed several tests on the reliability of the computed $H_\beta$ indices which have given encouraging results. For stars with well known stellar parameters, as is the case of Procyon and the Sun, the predicted indices are within the observational errors. For cool dwarfs the accuracy of the computed indices is within the error limits of the empirical relations. For stars hotter than 10000 K the differences

**Table 3.** Upper table: log $g$ (col. 7) from the observed $H_\beta$ index (col. 5) is compared with log $g$ (col. 6) of stars having either $T_{\rm eff}$ (col. 4) or log $g$ (col. 6) or both determined as fundamental values and taken from the literature (sources in col.9). Last column indicates which are the fundamental parameters. Lower table: log $g$ (col. 7) from the observed $H_\beta$ index (col. 5) is compared with log $g$ (col. 6) derived by Napiwotzki et al. (1993) (NSW93) from H$\beta$ profiles. The temperature $T_{\rm eff}$ given in col. 4 was derived by NSW93 from the Strömgren photometry.

| HR | Star | HD | $T_{\rm eff}$(K) | H[1]$_\beta$(obs) | log $g$[2] | log $g$($H_\beta$) | $\Delta$log $g$ | source[3] | fundamental |
|---|---|---|---|---|---|---|---|---|---|
| 472 | $\alpha$ Eri | 10144 | 14370±470 | 2.607±0.003 | 2.30±0.30ph | 2.64 | 0.34 | 2 | $T_{\rm eff}$ |
| 1728 | AR Aur A+B | 34364 | 10900±250 | 2.861±0.001 | 4.29±0.03 | 4.28 | -0.01 | 2 | log $g$ |
| 1790 | $\gamma$ Ori | 35468 | 20930±950 | 2.613±0.001 | 3.60 (3.60±0.1b, 4.0±0.5ph) | 3.67 | 0.07 | 2 | $T_{\rm eff}$ |
| 2004 | $\kappa$ Ori | 38771 | 25580±1290 | 2.566±0.022 | 3.30 (<3.30b,<3.5ph) | 3.39 | 0.09 | 2 | $T_{\rm eff}$ |
| 2088 | $\beta$ Aur A+B | 40183 | 9077±217 | 2.889±0.006 | 3.95±0.010 | 3.66 | -0.29 | 1 | $T_{\rm eff}$,log $g$ |
| 2294 | $\beta$ CMa | 44743 | 24020±1150 | 2.593±0.004 | 3.60 (3.60±0.1b,4.0±0.5ph) | 3.71 | 0.11 | 2 | $T_{\rm eff}$ |
| 2421 | $\gamma$ Gem | 47105 | 9220±330 | 2.865±0.009 | 3.50 (3.5±0.15b,3.5±0.5ph) | 3.48 | -0.02 | 3 | $T_{\rm eff}$ |
| 2491 | $\alpha$ CMa A | 48915 | 9940±210 | 2.907±0.003 | 4.33±0.05 | 4.24 | -0.09 | 3 | $T_{\rm eff}$,log $g$ |
| 2618 | $\epsilon$ CMa | 52089 | 20210±950 | 2.577 | 3.10 (3.1±0.1b,<3.0ph) | 3.01 | -0.09 | 2 | $T_{\rm eff}$ |
| 3335 | VV Pyx A+B | 71581 | 10200±500 | 2.881 | 4.09±0.01 | 4.13 | 0.04 | 2 | log $g$ |
| 3685 | $\beta$ Car | 80007 | 9150±240 | 2.836 | 3.50±0.5ph | 3.19 | -0.31 | 2 | $T_{\rm eff}$ |
| 3982 | $\alpha$ Leo | 87901 | 12090±370 | 2.723±0.006 | 3.60 (3.6±0.1b,3.2±0.3ph) | 3.51 | -0.09 | 2 | $T_{\rm eff}$ |
| 4662 | $\gamma$ Crv | 106625 | 12510±580 | 2.717±0.004 | 3.70 (3.7±0.1b,4.0±0.5ph) | 3.57 | -0.13 | 2 | $T_{\rm eff}$ |
| 4853 | $\beta$ Cru | 111123 | 26150±1110 | 2.596±0.002 | 4.20±0.5ph | 3.98 | -0.22 | 2 | $T_{\rm eff}$ |
| 5056 | $\alpha$ Vir A+B | 116658 | 23070±1000 | 2.605±0.004 | 3.84±0.17 | 3.79 | -0.05 | 2 | $T_{\rm eff}$,log $g$ |
| 5132 | $\epsilon$ Cen | 118716 | 24740±1210 | 2.608±0.001 | 4.50±0.5ph | 4.04 | -0.46 | 2 | $T_{\rm eff}$ |
|  | V 760 Sco A+B | 147683 | 15500 | 2.703±0.007 | 4.22±0.02 | 4.13 | -0.09 | 4,2 | log $g$ |
| 6414 | U Oph A | 156247 | 16500±500 | 2.695±0.001 | 4.08±0.02 | 4.22 | 0.14 | 2 | log $g$ |
| 6622 | V 539 Ara A | 161783 | 18000 | 2.665±0.003 | 3.941±0.017 | 4.06 | 0.12 | 4,2 | log $g$ |
| 6879 | $\epsilon$ Sgr | 169022 | 9420±240 | 2.778 | 4.50±0.3ph | 2.89 | -1.61 | 2 | $T_{\rm eff}$ |
| 7001 | $\alpha$ Lyr | 172167 | 9600±180 | 2.903 | 4.10 (4.00±0.1b,3.8±0.3ph) | 4.03 | -0.07 | 3 | $T_{\rm eff}$ |
| 7790 | $\alpha$ Pav | 193924 | 17640±790 | 2.660±0.007 | 3.50±0.5ph | 3.93 | 0.43 | 2 | $T_{\rm eff}$ |
| 8425 | $\alpha$ Gru | 209925 | 13910±590 | 2.722 | 4.00±0.5ph | 4.01 | 0.01 | 2 | $T_{\rm eff}$ |
| 153 | $\zeta$ Cas | 3360 | 20930 | 2.627±0.005 | 3.78 | 3.89 | 0.11 | 5 |  |
| 1641 | $\eta$ Aur | 32630 | 16890 | 2.683±0.002 | 4.07 | 4.13 | 0.06 | 5 |  |
| 4119 | $\beta$ Sex | 90994 | 14570 | 2.730±0.003 | 4.18 | 4.26 | 0.08 | 5 |  |
| 5191 | $\eta$ UMa | 120315 | 17320 | 2.694±0.001 | 4.28 | 4.36 | 0.08 | 5 |  |
| 6588 | $\iota$ Her | 160762 | 17480 | 2.661±0.001 | 3.82 | 3.92 | 0.10 | 5 |  |
| 7447 | $\iota$ Aql | 184930 | 13520 | 2.707±0.004 | 3.73 | 3.72 | -0.01 | 5 |  |
| 7906 | $\alpha$ Del | 196867 | 10950 | 2.796±0.005 | 3.85 | 3.74 | -0.11 | 5 |  |
| 8585 | $\alpha$ Lac | 213558 | 9530 | 2.908±0.003 | 4.11 | 4.06 | -0.05 | 5 |  |
| 8634 | $\zeta$ Peg | 214923 | 11330 | 2.768±0.003 | 3.69 | 3.66 | -0.03 | 5 |  |
| 8781 | $\alpha$ Peg | 218045 | 9810 | 2.838±0.003 | 3.54 | 3.57 | 0.03 | 5 |  |
| 8965 | $\iota$ And | 222173 | 11850 | 2.725±0.005 | 3.47 | 3.45 | -0.02 | 5 |  |
| 8976 | $\kappa$ And | 222439 | 11310 | 2.833±0.002 | 4.23 | 4.22 | -0.01 | 5 |  |

[1] Hauck& Mermilliod (1998) (see also footnote 6 in the text).
[2] "ph" indicates gravities derived from the spectrophotometry; "b" indicates gravities derived from Balmer profiles.
[3] Sources for $T_{\rm eff}$ and log $g$ in cols. 4 and 6:
(1) Smalley et al. (2002) (Setal02); (2) Smalley & Dworetsky (1995) (SD95); (3) Gardiner et al. (1999) (GKS99); (4) Moon & Dworetsky (1985) (MD85); (5) Napiwotzki et al. (1993) (NSW93).

between observed and computed indices is of the order 2.0%. It becomes less than 0.2% when averaged over all the considered stars. It has to be stressed that the uncertainties in the computed indices deduced from our analysis is strongly related with the accuracy of the stellar parameters adopted from the literature, but also with the capability of the ATLAS9 model atmospheres to predict the observations. A discussion on the physical, numerical, and computational limits of the ATLAS9 models can be found in Kurucz (2005). In particular, the treatment of convection in cool stars, the treatment of rotation in hotter stars and the use of the LTE hypothesis for all the stars may affect some computational results for the different spectral types.

## Appendix A: Data for computing response functions for $H_\beta$ indices

All the data used for computing the response functions needed to obtain the synthetic $H_\beta$ index are collected in Table A.1. Columns 1-4 list the transmission curve for the narrow filter 212 and the transmission curve for the wide filter 214 both reproducing the filter set from Crawford & Mander (1966); columns 5-6 list the 1P21 photomultiplier sensitivity; columns

7-8 list the reflectivity of aluminium; columns 9-10 list the atmospheric transmission. We took all these data from the Kurucz code beta.forcd (see footnote (1)).

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**Table A.1.** Data used for computing the response function for the $H_\beta$ indices calcuations

| $\lambda_{212}$(Å) | $S_{212}$ | $\lambda_{214}$(Å) | $S_{214}$ | $\lambda_{1P21}$ | $S_{1P21}$ | $\lambda_{Al}$ | $S_{Al}$ | $\lambda_{air}$ | $a_{air}$ |
|---|---|---|---|---|---|---|---|---|---|
| 4760 | 0.000 | 4640 | 0.000 | 3000 | 0.08 | 3000 | 0.82 | 3000 | 4.50 |
| 4765 | 0.000 | 4650 | 0.000 | 3100 | 0.22 | 3500 | 0.83 | 3200 | 1.30 |
| 4770 | 0.000 | 4660 | 0.002 | 3200 | 0.45 | 3800 | 0.84 | 3400 | 0.84 |
| 4775 | 0.001 | 4670 | 0.005 | 3300 | 0.68 | 4000 | 0.85 | 3600 | 0.68 |
| 4780 | 0.002 | 4680 | 0.007 | 3400 | 0.81 | 4500 | 0.86 | 3800 | 0.55 |
| 4785 | 0.003 | 4690 | 0.009 | 3500 | 0.90 | 5000 | 0.97 | 4000 | 0.46 |
| 4790 | 0.004 | 4700 | 0.011 | 3600 | 0.95 | 5550 | 0.88 | 4500 | 0.31 |
| 4795 | 0.006 | 4710 | 0.014 | 3700 | 0.97 | 6000 | 0.89 | 5000 | 0.23 |
| 4800 | 0.006 | 4720 | 0.019 | 3800 | 0.99 | | | 5500 | 0.195 |
| 4805 | 0.010 | 4730 | 0.026 | 3900 | 1.00 | | | 6000 | 0.170 |
| 4810 | 0.014 | 4740 | 0.036 | 4000 | 1.00 | | | | |
| 4815 | 0.019 | 4750 | 0.049 | 4100 | 0.99 | | | | |
| 4820 | 0.032 | 4760 | 0.064 | 4200 | 0.98 | | | | |
| 4825 | 0.056 | 4770 | 0.098 | 4300 | 0.95 | | | | |
| 4830 | 0.113 | 4780 | 0.138 | 4400 | 0.91 | | | | |
| 4835 | 0.173 | 4790 | 0.195 | 4500 | 0.87 | | | | |
| 4840 | 0.263 | 4800 | 0.274 | 4600 | 0.83 | | | | |
| 4845 | 0.315 | 4810 | 0.368 | 4700 | 0.77 | | | | |
| 4850 | 0.480 | 4820 | 0.473 | 4800 | 0.71 | | | | |
| 4855 | 0.578 | 4830 | 0.551 | 4900 | 0.65 | | | | |
| 4860 | 0.623 | 4840 | 0.604 | 5000 | 0.58 | | | | |
| 4865 | 0.525 | 4850 | 0.640 | 5100 | 0.52 | | | | |
| 4970 | 0.453 | 4860 | 0.664 | 5200 | 0.46 | | | | |
| 4875 | 0.285 | 4870 | 0.683 | 5300 | 0.40 | | | | |
| 4880 | 0.139 | 4880 | 0.698 | 5400 | 0.34 | | | | |
| 4885 | 0.086 | 4890 | 0.701 | 5500 | 0.29 | | | | |
| 4890 | 0.053 | 4900 | 0.701 | 5600 | 0.24 | | | | |
| 4895 | 0.032 | 4910 | 0.683 | 5700 | 0.20 | | | | |
| 4900 | 0.019 | 4920 | 0.645 | 5800 | 0.16 | | | | |
| 4905 | 0.016 | 4930 | 0.563 | 5900 | 0.13 | | | | |
| 4910 | 0.014 | 4940 | 0.428 | 6000 | 0.10 | | | | |
| 4915 | 0.011 | 4950 | 0.325 | | | | | | |
| 4920 | 0.009 | 4960 | 0.244 | | | | | | |
| 4925 | 0.007 | 4970 | 0.197 | | | | | | |
| 4930 | 0.005 | 4980 | 0.150 | | | | | | |
| 4935 | 0.002 | 4990 | 0.109 | | | | | | |
| 4940 | 0.000 | 5000 | 0.086 | | | | | | |
| 4945 | 0.000 | 5010 | 0.066 | | | | | | |
| 4950 | 0.000 | 5020 | 0.056 | | | | | | |
| | | 5030 | 0.051 | | | | | | |
| | | 5040 | 0.046 | | | | | | |
| | | 5050 | 0.041 | | | | | | |
| | | 5060 | 0.035 | | | | | | |
| | | 5070 | 0.030 | | | | | | |
| | | 5080 | 0.025 | | | | | | |
| | | 5090 | 0.020 | | | | | | |
| | | 5100 | 0.015 | | | | | | |
| | | 5110 | 0.010 | | | | | | |
| | | 5120 | 0.005 | | | | | | |
| | | 5130 | 0.000 | | | | | | |
| | | 5140 | 0.000 | | | | | | |